

# Dissipation and displacement of hotspots in reaction-diffusion models of crime

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Edited by Luc E. Anselin, Arizona State University, Tempe, AZ, and approved January 5, 2010 (received for review September 22, 2009)

**The mechanisms driving the nucleation, spread, and dissipation of crime hotspots are poorly understood. As a consequence, the ability of law enforcement agencies to use mapped crime patterns to design crime prevention strategies is severely hampered. We also lack robust expectations about how different policing interventions should impact crime. Here we present a mathematical framework based on reaction-diffusion partial differential equations for studying the dynamics of crime hotspots. The system of equations is based on empirical evidence for how offenders move and mix with potential victims or targets. Analysis shows that crime hotspots form when the enhanced risk of repeat crimes diffuses locally, but not so far as to bind distant crime together. Crime hotspots may form as either supercritical or subcritical bifurcations, the latter the result of large spikes in crime that override linearly stable, uniform crime distributions. Our mathematical methods show that subcritical crime hotspots may be permanently eradicated with police suppression, whereas supercritical hotspots are displaced following a characteristic spatial pattern. Our results thus provide a mechanistic explanation for recent failures to observe crime displacement in experimental field tests of hotspot policing.**

crime pattern formation | hotspot policing | mathematical modeling | nonlinearity | partial differential equations

Crime is a ubiquitous feature of all modern cities, but not all neighborhoods are affected equally. In fact, serious crimes ranging from residential burglary to homicide are strongly patterned in time and space, forming crime “hotspots” (1–3). Studies show that policing actions directed at crime hotspots do lead to real reductions in offending and calls to the police for service (4, 5), while displacement of crime to adjacent settings may be less common than once thought (6–8). However, further gains in crime reduction are dependent upon gaining a quantitative understanding of the mechanisms that drive the emergence, spread, and dissipation of crime hotspots. Reaction-diffusion models, in which activators and inhibitors move, mix, and interact, provide a useful framework in which to investigate the formation of crime patterns and the impact of alternative policing strategies on crime hotspot stability. In this context, motivated offenders (activators) search their environment for suitable targets or victims (activators), which may also be mobile, following simple behavioral routines (9, 10). If an offender encounters a target in the absence of an effective security measure (inhibitor), then he is free to exploit that target. The immediate presence of security such as law enforcement is sufficient to deter that crime. Here we show that large-scale spatial crime patterns, including the formation of stationary crime hotspots, are strongly dependent upon the local diffusion of risk, driven by offender mobility in the environment, coupled with the phenomena of repeat and near repeat victimization.

## Model

We study a reaction-diffusion system involving mobile criminal offenders within a square environment with periodic boundary conditions (11). Potential crime targets such as homes, automobiles, or persons, depending on crime type, are continuously dis-

tributed in space, and each location  $\mathbf{x} = (x, y)$  is characterized by a risk of victimization, defined as a field  $A(\mathbf{x}, t)$ , representing general environmental cues about the feasibility of committing a successful crime (12–15) and/or specific knowledge offenders possess about target or victim vulnerability in the area (16–18). While  $A(\mathbf{x}, t)$  is easiest to conceptualize in reference to stationary targets (such as homes in the case of burglary), it may also be used to represent the risk of attacks on mobile victims at any given spatial location (19, 20). Our model and its results are therefore independent of crime type.

Risk is the sum of a fixed component  $A^0(\mathbf{x})$ , which is stationary in time but potentially variable in space, and a dynamic component  $B(\mathbf{x}, t)$ , which evolves in time according to

$$\frac{\partial B}{\partial t} = \eta D \nabla^2 B - \omega B + \kappa \rho A, \quad [1]$$

where  $\rho(\mathbf{x})$  is the density of criminal agents. The parameter  $\kappa$  measures the growth in risk at location  $\mathbf{x}$  given crimes occurring there at a rate per unit area  $\rho A$ . Thus,  $\kappa$  is an attractive force pulling offenders back to locations where they have successfully committed crimes, a dynamic inferred from empirical evidence for repeat crimes being concentrated in time shortly after the initial event (Fig. 1A) (*SI Empirical Crime Data*) (21–23). The parameter  $\omega$  determines the rate at which the elevated risk decays towards the fixed environmental value  $A^0(\mathbf{x})$ , which generally occurs within days to weeks depending upon crime type (Fig. 1A). Finally,  $D$ , a diffusion coefficient, and  $\eta \in [0, 1]$  control the rate of diffusive spread of crime risk within the local environment, describing a so-called “near repeat” phenomenon whereby targets within several hundred meters of an initial crime are more likely to be victimized than by chance (Fig. 1B) (19, 21, 24).

We assume that offenders search for criminal opportunities in the local environments surrounding the key activity nodes in their daily routines; these may include home, work, or recreation sites (Fig. 1C) (25–27). Offenders also preferentially select vulnerable targets, i.e., those with high associated risk  $A(\mathbf{x}, t)$ , usually to minimize the danger of capture or confrontation (16, 17, 28, 29). Thus the density of criminal offenders  $\rho$  at a spatial location  $\mathbf{x}$  evolves according to

$$\frac{\partial \rho}{\partial t} = D \vec{\nabla} \cdot [\vec{\nabla} \rho - 2\rho \vec{\nabla} \ln A] - \rho A + \gamma, \quad [2]$$

where  $\vec{\nabla}$  is the gradient operator. Offenders move up gradients of  $\ln A$ , but simply diffuse in the absence of a risk gradient. They

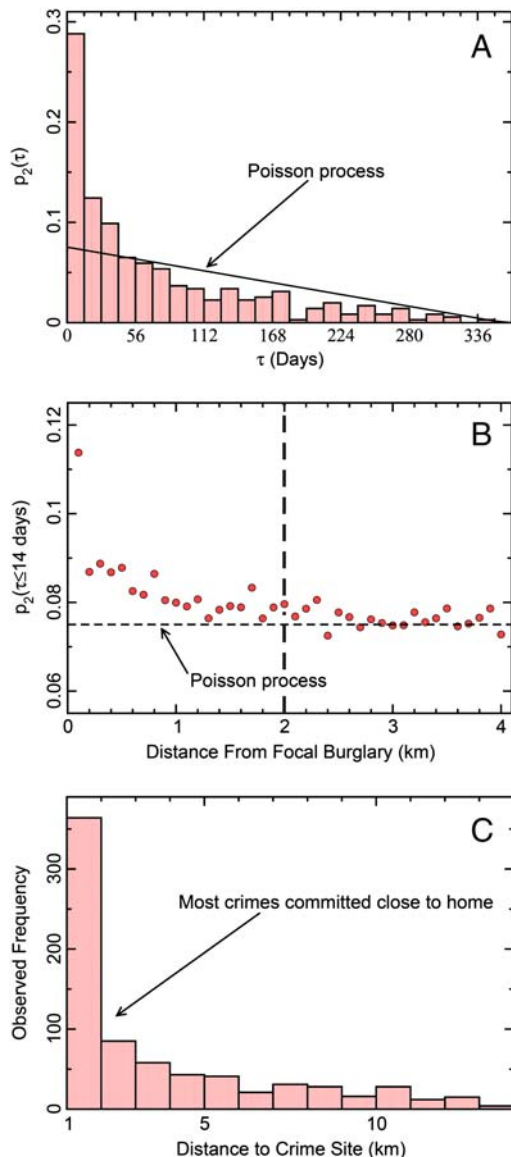
Author contributions: M.B.S., P.J.B., A.B., and G.T. designed research; M.B.S., P.J.B., A.B., and G.T. performed research; M.B.S. and A.B. contributed new analytic tools; M.B.S., P.J.B., A.B., and G.T. analyzed data; M.B.S., P.J.B., A.B., and G.T. wrote the paper.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

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This article contains supporting information online at [www.pnas.org/cgi/content/full/0910921107/DCSupplemental](http://www.pnas.org/cgi/content/full/0910921107/DCSupplemental).



**Fig. 1.** Spatio-temporal patterns of criminal behavior. Criminal offenders are more likely to return to the same and/or nearby locations to commit repeat crimes. Here we illustrate this fact through the specific example of single-family residential burglary, using data from Long Beach, California, over the years 2000–2005 (*SI Empirical Crime Data*). (A) The probability of observing a time separation of  $\tau$  days between events at individual residences burgled exactly twice within a fixed temporal window of  $D = 364$  days. We compare the observed distribution with an expected distribution of repeat crimes assuming they are Poisson distributed in time:  $p_2(\tau) = 2(D - \tau) / D(D + 1)$  (21). Repeat burglaries are much more likely to occur in the four weeks following the initial event than by chance, suggesting that burglars are biased to return to the same locations because of unique environmental cues and/or specific knowledge gained in the first burglary. (B) Repeat burglary probability in the two weeks following an initial crime as a function of distance from the initial crime. The Poisson expectation is calculated as  $p_2(\tau \leq 14) = \sum_{\tau=1}^{14} 2(D - \tau) / D(D + 1)$ . Burglaries occurring within two weeks of one another separated by more than approximately 2000 m are statistically independent (Poisson), whereas those that are closer to one another in space show strong spatial dependency; the interpretation is that the initial burglary increases the likelihood that neighbors are also victimized. Crime risk therefore spreads spatially up to distances  $\leq 2000$  m (dashed line). Similar repeat and near-repeat victimization dynamics are recognized for shootings (19) and insurgent attacks (23), indicating that these are generic features of a wide range of crime types. (C) Journey-to-crime distribution for residential burglars measured as the distance in kilometers from the offender's residence to the site of a burglary. Most offenders search for targets in the immediate vicinity of home, a key activity node in their daily routine.

thus seek out and victimize targets at greater risk whenever such differences are present in the environment. Offenders also exit the system at rate  $\rho A$ , following commission of crimes, and are introduced at a constant, but possibly spatially variable, rate per unit area  $\gamma$ .

## Results

Eqs. 1 and 2 are coupled partial differential equations (PDEs) describing the space-time coevolution of crime risk and offender populations. We are interested in the conditions that cause the risk of crime,  $B$ , to be nonuniformly distributed in space. Linear stability analysis shows in fact that crime risk will form dense, well-spaced hotspots whenever the diffusion of the risk produced by individual crimes is spatially broad enough to bind local crimes together, but not so extensive that distant crimes are also connected (11). A schematic illustration of the mechanism driving crime pattern formation is shown in Fig. 2. Individual crimes are uniquely associated with a local spatial region conceptually similar to a Voronoi polygon, an area parametrically defined as  $C\omega/\gamma$  (Fig. 2A). The inverse of this term is the average density of crime. The diffusion of risk from focal crime sites to nearby spatial locations defines a second spatial region that we term the “area of influence” of a crime given by  $\eta\kappa/\omega$  (see Fig. 1B). Crime hotspots will not emerge if the areas of influence of individual crimes do not overlap and therefore do not form risk clusters (i.e.,  $\eta\kappa/\omega < C_f\omega/\gamma$ ) (Fig. 2A), or conversely, if the areas of influence of individual crimes overlap with many others spreading risk evenly through the urban environment (i.e.,  $\eta\kappa/\omega > C_h\omega/\gamma$ ) (Fig. 2B). Thus, crime hotspots will emerge whenever  $C_f\omega/\gamma < \eta\kappa/\omega < C_h\omega/\gamma$  (Fig. 2C). The conditions necessary for crime hotspots to form hinge on the geographic nature of offender foraging behavior and underscore the importance of spatially restricted offender search in spreading risk in local neighborhoods (see Fig. 1C) (29, 30). Numerical integration of Eqs. 1 and 2 (see *SI Methods*) confirms that hotspots do not form when the diffusion of risk is either spatially restricted or spatially expansive (Fig. 2D) but do emerge with a distinct spatial scale set by the maximally unstable wavelength  $\lambda_* = 2\pi/|k_*|$  otherwise (Fig. 2E).

Weakly nonlinear analysis shows, however, that crime hotspots may arise as both supercritical and subcritical bifurcations, or large qualitative shifts from the homogeneous steady state (Fig. 3A). Supercritical bifurcations ( $\epsilon > 0$  in Fig. 3A) see the nucleation and growth of crime hotspots from small spikes in crime due to the linear instability of the system, whereas subcritical bifurcations ( $\epsilon < 0$  in Fig. 3A) are the result of large local spikes in crime that override linear stability. Referring to Fig. 2A, for example, crimes are sufficiently spaced such that the area of influence of a single crime generally does not overlap with that of others and the system is linearly stable with a spatially uniform crime distribution. If one location were to experience a sudden and extreme surge in crime, however, then the increased risk may suddenly spill over into the area of influence of an adjacent crime binding those areas together to create a subcritical hotspot. Alternatively, even where all crimes are bound together by the broad spatial diffusion of risk (Fig. 2B), a large surge in crime at one location may create a nucleation point pulling offenders away from a uniform distribution and causing a subcritical crime hotspot to emerge.

One of our key results is that subcritical hotspots may form under a wide array of conditions, paralleling those that form supercritical hotspots (Fig. 3B). The parameter combination  $\eta B$  describes the dynamic component of crime risk, approximately the probability of repeat victimization occurring at any given location. The variable  $\eta A^0$  describes the fixed environmental risk of crime, approximately the probability of crimes occurring independently in space and time as a result of a stationary Poisson process. Small values of  $\eta A^0$  describe environments characterized







One possible explanation for the infrequent observation of displacement in empirical settings is that environments are sufficiently heterogeneous to limit the feasibility of offenders moving from favored habitats to adjacent areas that may be bereft of targets or victims, or may experience much higher levels of surveillance (36). Our analysis is based on a very regular, homogeneous environment where the baseline crime risk is uniformly distributed and thus displacement is not constrained by environmental structure. The assumption of environmental homogeneity could be perceived as a weakness of the modeling approach since most, if not all, real-world environments will be heterogeneous in the distribution of background crime risk (18, 37). We suggest, however, that a homogeneity assumption is useful as a theoretical baseline precisely because it is difficult to find in real-world settings. Were real-world environments as homogeneous as rendered in the current model, displacement would perhaps be much more common.

It is also possible that crime displacement has not been observed in empirical settings because controlled experiments have looked for only immediate spatial displacement in one to two block catchment areas surrounding crime suppression sites (8, 32). Displacement of supercritical hotspots in our continuum models is generally to a location midway between the hotspots being suppressed, effectively  $\lambda_*/2$ , suggesting that the concern in empirical studies should be with crime displacement over intermediate distances (38).

Finally, the rarity of displacement in empirical tests of hotspot policing may mean that most real-world hotspots are subcritical rather than supercritical. However, we might also expect different crime types to generate different types of hotspots and therefore respond differently to hotspot policing actions. For example,

open-air drug markets (7, 39) may require a large, initial spike in drug transactions involving multiple sellers to become established in a fixed spatial location. The failure of drug markets to reemerge following suppression, despite continued small-scale transactions on the street (40), is behavior consistent with classification of these markets as subcritical hotspots. By contrast, residential burglary or auto theft may be more likely to nucleate into supercritical hotspots, since these crimes are most often committed by either individuals acting alone or small co-offending groups (41). Displacement may be expected for these crime types since the offenders involved may be better able to respond to suppression by searching for targets in adjacent areas. However, support for such distinctions will require additional field experimentation.

We have shown that PDE reaction-diffusion models provide a mechanistic explanation for crime pattern formation given simple assumptions about the diffusion of crime risk and localized search by offenders. These models result in subtle observations about the fundamental dynamics of crime hotspots that may not be obtained through direct empirical study of crime data. The differences between supercritical and subcritical crime hotspots, for example, explain different outcomes of hotspot policing, including both hotspot suppression and displacement, and suggest that policing strategies need to be tailored to hotspot type.

**ACKNOWLEDGMENTS.** This work was supported in part by the US National Science Foundation Human Social Dynamics Program (BCS-0527388), NSF Mathematics (DMS-0907931), Office of Naval Research (N000141010221), and Army Research Office (W911NS-09-1-0559 and 50363-MA-MUR). We thank the Long Beach and Los Angeles Police Departments for their support.

- Brantingham PJ, Brantingham PL (1984) *Patterns in Crime* (Mackmillan, New York).
- Chainey S, Ratcliffe J (2005) *GIS and Crime Mapping* (John Wiley & Sons, Chichester).
- Eck J, Weisburd D (1995) Crime places in crime theory. *Crime and Place*, ed Eck J (Criminal Justice Press, Monsey), pp 1–33.
- Braga AA (2001) The effects of hot spots policing on crime. *Ann Amer Acad Polit Soc Sci* 578:104–125.
- Braga A (2005) Hot spots policing and crime prevention: A systematic review of randomized controlled trials. *J Exp Criminol* 1:317–342.
- Green L (1995) Cleaning up drug hot spots in Oakland, California: The displacement and diffusion effects. *Justice Q* 12:737–754.
- Taniguchi TA, Rengert GF, McCord ES (2009) Where size matters: Agglomeration economies of illegal drug markets in Philadelphia. *Justice Q* 26:670–694.
- Weisburd D, et al. (2006) Does crime just move around the corner? A controlled study of spatial displacement and diffusion of crime control benefits. *Criminology* 44:549–592.
- Cohen LE, Felson M (1979) Social-change and crime rate trends—routine activity approach. *Am Sociol Rev* 44:588–608.
- Sherman LW, Gartin PR, Buerger ME (1989) Hot spots of predatory crime: routine activities and the criminology of place. *Criminology* 27:27–55.
- Short MB, et al. (2008) A statistical model of criminal behavior. *Math Models Methods Appl Sci* 18:1249–1267.
- Keizer K, Lindenberg S, Steg L (2008) The spreading of disorder. *Science* 322:1681–1685.
- Wilson JQ, Kelling G (1982) Broken windows: The police and neighborhood safety. *The Atlantic Monthly* 127:29–38.
- Sampson RJ, Raudenbush SW, Earls F (1997) Neighborhoods and violent crime: A multi-level study of collective efficacy. *Science* 277:918–924.
- Skogan WG (1990) *Disorder and Decline: Crime and the Spiral of Decay in American Neighborhoods* (University of California Press, Berkeley).
- Wright RT, Decker SH (1994) *Burglars on the Job: Streetlife and Residential Breakins* (Northeastern University Press, Boston).
- Wright RT, Decker SH (1997) *Armed Robbers in Action: Stickups and Street Culture* (Northeastern University Press, Boston).
- Johnson S (2008) Repeat burglary victimisation: A tale of two theories. *J Exp Crim* 4:215–240.
- Ratcliffe JH, Rengert GF (2008) Near-repeat patterns in Philadelphia shootings. *Security J* 21:58–76.
- Tita G, Griffiths E (2005) Traveling to violence: The case for a mobility-based spatial typology of homicide. *J Res Crim Delinq* 42:275–308.
- Short MB, D'Orsogna MR, Brantingham PJ, Tita G (2009) Measuring and modeling repeat and near-repeat burglary effects. *J Quant Criminol* 25:325–339.
- Farrell G, Pease K (1993) *Once Bitten, Twice Bitten: Repeat Victimization and Its Implications for Crime Prevention* (Home Office, London).
- Townsley M, Johnson SD, Ratcliffe JH (2008) Space time dynamics of insurgent activity in Iraq. *Security J* 21:139–146.
- Townsley M, Homel R, Chaseling J (2003) Infectious burglaries: A test of the near repeat hypothesis. *Brit J Criminol* 43:615–633.
- Brantingham PL, Brantingham PJ (1991) Notes on the geometry of crime. *Environmental Criminology*, ed Brantingham PJ (Waveland Press, Prospect Heights), 2nd Ed, pp 27–54.
- Gonzalez MC, Hidalgo CA, Barabasi AL (2008) Understanding individual human mobility patterns. *Nature* 453:779–782.
- Rengert GF, Piquero AR, Jones PR (1999) Distance decay reexamined. *Criminology* 37:427–445.
- Cornish DB, Clarke RV (1987) Understanding crime displacement: An application of rational choice theory. *Criminology* 25:933–947.
- Bernasco W, Nieuwbeerta P (2005) How do residential burglars select target areas? A new approach to the analysis of criminal location choice. *Brit J Criminol* 45:296–315.
- Block R, Galary A, Brice D (2007) The journey to crime: Victims and offenders converge in violent index offences in Chicago. *Security J* 20:123–137.
- Rossmo DK (2000) *Geographic Profiling* (CRC Press, Boca Raton).
- Braga AA, Bond BJ (2008) Policing crime and disorder hot spots: A randomized controlled trial. *Criminology* 46:577–607.
- Sherman L, Rogan D (1995) Deterrent effects of police raids on crack houses: A randomized controlled experiment. *Justice Q* 12:755–782.
- Repetto TA (1976) Crime prevention and displacement phenomenon. *Crim Delinq* 22:166–167.
- Brantingham PJ, Brantingham PL (2003) Anticipating the displacement of crime using the principles of environmental criminology. *Crime Prevention Studies* 16:119–148.
- Rengert G, Ratcliffe JH, Chakravorty S (2005) *Policing Illegal Drug Markets: Geographic Approaches to Crime Reduction* (Criminal Justice Press, Monsey, NY).
- Brantingham PJ, Brantingham PL (1981) *Environmental Criminology* (Sage, Beverly Hills).
- Barr R, Pease K (1990) Crime placement, displacement, and deflection. *Crime & Justice: A Review of Research*, ed Tonry M (University of Chicago Press, Chicago), 12, pp 277–318.
- Weisburd D, Green L (1995) Policing drug hot spots: The jersey city drug market analysis experiment. *Justice Q* 12:711–736.
- Kennedy D (2009) *Deterrence and Crime Prevention: Reconsidering the Prospect of Sanction* (Routledge, London).
- Reiss AJ, Farrington DP (1991) Advancing knowledge about co-offending: Results from a prospective longitudinal survey of London males. *J Criminal Law Criminol* 82:360–395.