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The Differential Calculus of Consent

Abstract - Existing treatments of the choice of an optimal voting rule ignore the effects of the rule on political bargaining. Specifically, more stringent majority requirements reduce intra-coalitional free riding in political compromise, leading to greater gains from political trade. Once this benefit of increasing the vote share necessary to enact a proposal is recognized, we suggest that the optimal voting rule in the presence of transactions costs will actually be closer to unanimity than the optimal majority derived by Buchanan - Tullock [1962].

Keywords – Free-riding, voting, constitutional design, political coalitions

JEL classification codes – D60, D71, H11

1. – *Introduction*

In the foundational work on constitutional economics, Buchanan – Tullock [1962] provided a framework for determining the optimal majority size required for collective action. In their set-up, a rational agent who is unaware of his eventual policy preferences chooses the majority size that minimizes the expected costs imposed on him through collective action. In the absence of transactions costs in reaching a decision, Buchanan and Tullock determine that unanimity yields the cost minimizing result because it is the only voting rule ensuring that uncompensated external costs are not imposed on an individual. While appreciating the attractive properties this unanimity rule provides by protecting the minority from being exploited by the majority, Buchanan and Tullock recognize the likelihood of strategic actions on the part of voters under a unanimity system, suggesting that the optimal voting proportion will fall short of unity.

Recently, however, Guttman [1998] has suggested that Buchanan and Tullock ignored one of the important costs related to a more stringent voting rule. That is, they did not consider the opportunity costs of rejected proposals that the individual favors. Guttman suggests that, once these oppor-

tunity costs are considered, Buchanan and Tullock's support for unanimity falls apart.

Guttman's result, however, while correct on its own terms, hinges on its own simplification of the collective decision making process. By disallowing logrolling of any sort, it is impossible for these external opportunity costs to be internalized in the political process. As is implied in Buchanan's [1998] and Tullock's [1998] responses to Guttman, the reasons given for ignoring the possibility of logrolling are suspect, and a rich analytical account of the process of choosing the decision rule must include some sort of trading as a possibility.

In this article, we augment the classic Buchanan and Tullock analysis, recognizing the opportunity costs of proposals foregone, under more realistic assumptions about the market for votes. In Section 2, we draw on Parisi's [1998 and forthcoming] insight regarding free riding in the logrolling context, as well as Riker's [1962] size principle for political coalitions, to discuss how the amount of trading in the political market will be endogenous to the given voting rule. In Section 3, we incorporate this analysis in the Buchanan and Tullock framework to suggest that incorporating the opportunity cost of foregone proposals might actually yield a stricter voting rule than the one implied by *The Calculus of Consent*.

2. – *Trading in the political market is endogenous to the voting rule*

Parisi [forthcoming] presents a formal model of Coasian bargaining in the legislative context in which, as long as the bargains are enforceable and side payments are allowed, the eventual proposal adopted by the majority coalition is independent of the voting rule used. That is, in the case of zero transactions costs, the Coase [1960] theorem applies in the political market. This implies that all gains from trade will be exploited and the initial assignment of property rights does not affect the eventual proposal adopted by the voters. Thus, dictatorship, majority rule, and unanimity are all equivalent in terms of the efficiency of their eventual outcomes. Parisi shows how trading in this context will lead to social welfare maximization with respect to the voters' utilities.

According to Parisi's 'Political Coase Theorem', if political bargains are attainable at no cost and political agreements are enforceable, the resulting political equilibrium will be unique and will occur at a point of social maximum. Any point other than the global maximum will be unstable, as there will always be enough surplus to allow for side payments to voters in exchange for policy concessions. Once the socially optimal point is reached, there will be no opportunity to destabilize the policy arrangement. In ef-

fect, Arrow's [1963] result is avoided through the introduction of cardinal preferences through the political bargaining.

However, once transactions costs are incorporated, the Coase theorem can no longer be invoked and discussion of the optimal voting rule is no longer pointless. Of course, in their classic treatment of these issues, Buchanan and Tullock recognized this effect of transaction costs, as did Wicksell [1896] before them, suggesting that the hold-out problem generates decision costs that must be included in a representative voter's decision regarding the optimal majority size in the constitutional stage. Specifically, the larger the proportion of voters required to take collective action, the higher the cost of this hold-out problem because each individual's incentive to try to capture all of the surplus for himself increases, at an increasing rate, as the voting rule approaches unanimity. In their words, «Since each voter, then, has a monopoly on an essential resource (that is, his consent), each person can aim at obtaining the entire benefit of agreement for himself» [BUCHANAN-TULLOCK, 1962]. This recognition caused Buchanan and Tullock to temper their support for relatively stringent majority requirements.

Parisi's [forthcoming] analysis suggests, however, that there is another kind of strategic transaction cost that comes into play in this context. Within any majority coalition, the incentive to free ride on other coalition members' bargaining efforts decreases as the voting rule approaches unanimity. If the object of one individual's bargaining generates benefits for other individuals who are not involved in the bargain, the bargaining generates positive externalities. Because of this, the incentive to initiate the bargaining will be diminished as every coalition member desires to free ride off of the efforts of the others, generating a sub-optimal level of bargaining for the common interest [PARISI, 1995]. Because of this, less trading will occur. However, as the proportion of voters required for passage increases, where in the limit of unanimity every voter has an effective veto over any proposal, this incentive to free ride decreases.

The collective action problem described above is not different from any other free riding problem in a Coasian setting. Dixit - Olson [2000] have discussed the collective action problem in the context of Coasian bargaining, questioning the practical validity of the Coasian proposition in a multi-party context. If the object of one individual's bargaining generates a benefit to other individuals who are not involved in the bargain, what is obtained through the bargaining of one individual creates a positive externality to other individuals. Thus the incentives to undertake the bargaining may be seriously undermined. Every individual wishes to be the free rider, having somebody else pay for the common good. Thus, similar to any public good situation, there will be a sub-optimal level of bargaining for the common interest.

As is suggested by Parisi's model, Guttman [1998] is right in claiming that the Coase Theorem implies a certain irrelevance of the voting rule. That is, with unfettered trading, all value producing moves will be made regardless of the decision rule. Thus, any discussion of an optimal voting rule must invoke some transaction cost-based limitation on trade to avoid being entirely moot. However, in any collective action situation involving a policy that can take on continuous values, there must be some bargaining by virtue of the fact that individuals do not have identical preferences. Unless individuals are truly homogeneous, there must be some compromise among the members of a coalition supporting a given proposal, otherwise, for every conceivable issue, there are likely to be as many proposals as there are voters.

There then exists a tension that Buchanan – Tullock [1962] left implicit. Clearly there is some degree of trading but, at some point, transactions costs stop the trading before all gains are exploited. The relevant question to address then is how majority coalitions form in the first place.

Riker's [1962] size principle gives some insight[1] regarding the formation of legislative coalitions. According to Riker, «participants create coalitions just as large as they believe will ensure winning and no larger». That is, we should expect legislative coalitions to be minimum winning coalitions[2]. The intuition of this claim is quite clear. The coalition obviously needs to secure victory for its legislative bundle, but bringing in new voters to the coalition will require concessions on the part of existing coalition members. Thus, limiting the extent of their concessions conditional on securing passage of the proposal implies that coalition members will favor the smallest coalition possible in which their legislative bundle is selected by the legislature.

Even in situations where Riker's argument does not formally hold (specifically, in non-zero sum games), the intuition is suggestive and consistent with empirical observation. There are costs involved in bringing additional voters into the coalition. Prior to reaching a winning proportion of voters, the value of the legislative bundle (L)[3] to the coalition is zero since there are not enough votes to pass (L). However, once the coalition is sufficiently large to secure victory, inclusion of new coalition members adds nothing to L's prospects of passage. Logrolling opportunities with additional coalition

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[1] Riker's [1962] size principle, as he modeled it, only applies to zero sum games, however, the concept is nonetheless suggestive of how coalitions form even in non-zero sum games.

[2] Weingast [1979] provides a model in which we should expect more inclusive coalitions even under a majority rule system. For a discussion of the arguments both favoring and criticizing the size principle, see Stratmann [1997].

[3] L is used to refer to the legislative bundle supported at a given moment in time by the coalition.

members would add nothing to the expected utility of the majority coalition members, thus eliminating any incentive to extend membership beyond the minimum winning coalition. Only actual side payments in a currency different from the policy dimensions (*e.g.* cash payments) could induce incumbent majorities to extend membership to their coalition beyond the minimum winning coalition. But such option is generally not viable in real world politics. This intuition is also consistent with the empirical observation that coalition governments in a multi-party system rarely reach beyond the minimum required majority.

Assuming that the coalition agreement is binding and enforceable, to make it worthwhile for the winning coalition to let another member into its ranks, the new member must be able to offer a side payment large enough to compensate the coalition for a movement from L to L'. On the other hand, if the coalition supporting L does not have enough votes for passage, the coalition is willing to pay some of its surplus to secure additional members to ensure passage, which it will do by purchasing the support of voters whose policy preferences lie closest to L.

The dynamics of these situations present, respectively, hold out problems and free rider problems. In the case where an outsider is looking to buy his way into the coalition, any member of the coalition holds a veto over his entrance and has the incentive to hold out to receive the entire payment. In the situation where the coalition needs to purchase more support, each member of the coalition has the incentive to free ride on the concessions of other coalition members. Within the existing coalition, the transactions costs presented by the hold out problem are independent of the voting rule[4]. The free rider problem, however, as suggested by Parisi is dependent on how large a majority is required. Free riding is mitigated as the size of the required majority rises.

Clearly then, the size of a winning legislative coalition will be endogenous to the size of the majority needed for passage of legislation. In the extreme case of unanimity, a minimum-winning coalition must, of necessity, include the entire legislature. But beyond that trivial insight, this discussion suggests that a higher required majority will induce more political bargaining, with greater potential for the exploitation of gains from political trades.

This endogeneity of political bargaining suggests that Guttman's [1998] analysis is overly restrictive and ignores an important consideration in an individual's choice of the optimal voting rule. The rational individual will recognize that as the size of the required majority increases, decision costs will

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[4] In some sense, it is irrelevant whether you have 1 holdout or 100 holdouts. In either case, trading will be stifled.

also increase due to the hold out problem. This was clearly laid out in Buchanan and Tullock's graphical representation of the choice of the voting rule. However, an increased majority size ameliorates free riding within the political coalition, inducing bargaining and providing a representative agent with an expected marginal gain. Because the total value of the surplus increases as political trading increases, a representative individual's welfare increases as trade increases. Thus, an individual will recognize these benefits from decreased free riding within political coalitions and will incorporate them in his calculus. In essence, Guttman's result should be revisited in light of the fact that individuals may actually expect positive incremental benefits as the required majority increases in size.

3. – *The optimal voting rule*

In the seminal Buchanan and Tullock treatment, the representative voter, who is ignorant of his eventual preferences, chooses the voting rule that minimizes the sum of the expected external costs imposed on him by a majority (C) and transactions costs (D). These external costs are decreasing as unanimity is approached, while transactions costs are increasing (Figure 1).

Figure 1 – *The calculus of consent*
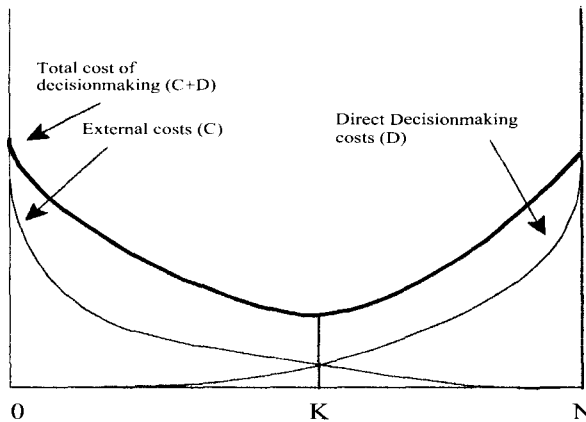
The endogenous political bargaining argument suggests the addition of an expected benefits curve to the standard Buchanan and Tullock analysis. The choice of optimal voting rules should indeed account for the effects induced by alternative voting rules on the vote trading equilibrium discussed

above. Greater majority requirements increase the minimum winning coalition and consequently increase the scope (and expected benefits) of inter-coalition bargaining. In the context of Buchanan and Tullock's graphical representation of alternative voting rules, the endogenous bargaining curve should be included in the calculus of consent in terms of cost of forgone surplus from political trade (*F*). An increase in the required majority will reduce such costs, by forcing cooperation and bargaining among coalition members (Figure 2).

Figure 2 – *The differential calculus of consent*
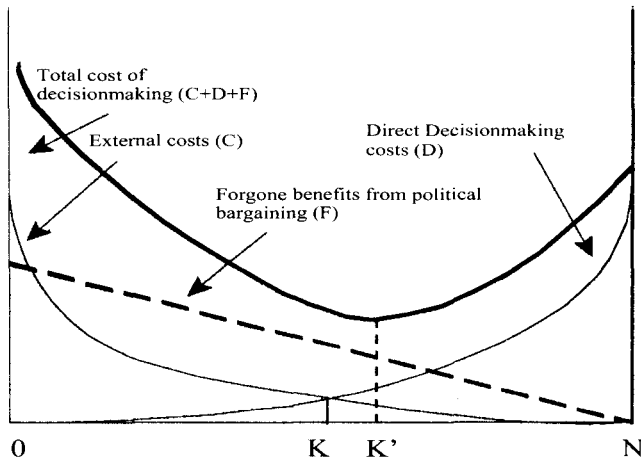
It then follows, in the individual's maximization of his expected gain, the optimal choice would fall on the voting rule where the summation of the three cost curves is minimized (*K'*). Given the negative slope of forgone surplus curve (*F*), *K'* will necessarily lie closer to unanimity than the optimal voting rule (in the presence of transaction costs) as Buchanan and Tullock presented it.

4. – *Conclusion*

This paper revisits the debate on optimal voting rules in the light of recent critiques of the classic analysis of Buchanan and Tullock. Recently, Guttman argued that Buchanan and Tullock neglected to account for the opportuni-

ty costs of proposals that are rejected by a super-majority rule, despite these proposals being efficient in the sense of Kaldor and Hicks. Taking these opportunity costs into account would argue for a less inclusive decision rule (*i.e.*, a smaller critical proportion of votes) than the rule proposed by Buchanan and Tullock.

There is a fundamental tension in the literature on setting optimal voting rules. If there is no impediment to political bargaining, the voting rule is irrelevant in terms of the final outcome, since all gains from trade will be exhausted as the Coase Theorem would imply. On the other hand, few would suggest that no political bargaining takes place. Indeed, in a world of heterogeneous voters and continuous policy outcomes, bargaining arises almost by necessity. An important issue then involves what determines the point at which bargaining ceases. As with the Coase Theorem itself, transactions costs provide the answer. Previous work has focused only on the transactions costs of inter-coalitional bargaining, but we argue that intra-coalitional transactions costs must also be considered in determining the optimal majority size.

In this paper we consider a related dimension of the calculus of consent, unveiling the full implications of the free-rider problem whenever a coalition of voters tries to gather compensatory side-payments in order to add support for their proposals. A more complete analysis would take account of the free-rider problem for those proposals for which side-payments can be made. It is clear that the degree of legislative bargaining will be endogenous to the voting rule used for collective decisions. This is implied by both Riker's [1962] size principle and our discussion of free riding in the logrolling context. Any discussion of the optimal majority size must recognize this endogeneity. To that end, in this paper, we suggest that incorporating this insight presents a case for stricter majority requirements in the constitutional setting even in the face of costs of decision making.

Naturally, these results can be easily reconciled with the traditional wisdom, in that they merely call for a redefinition of the critical variables of the calculus of consent. Put differently, the different formulation offered in this paper does not change the nature of the Buchanan-Tullock result: there are types of costs which decrease and others that increase with the size of the required majority, and there is no a priori presumption that the minimum of the sum occurs at the majority rule. This is an entirely qualitative result, which calls for the explicit incorporation of such bargaining externalities, were they to exist, into the costs of decision making. While theoretical analysis can explain why these costs increase with the size of the majority, our qualitative results require some empirical examination of how fast these costs increase with the size of the majority.

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