Problem Set 1
Finance
February 4, 2005

1. You get a job as a junior associate in a Miami firm. You have no savings but would like to buy an expensive house so as to live the life of a lawyer. The cheapest suitable house you can find costs $500,000. Due to the expectation that you have a lucrative career ahead of you, a bank is willing to loan you the entire amount needed to buy your house (assume no closing costs, etc) at an interest rate of 8 percent (with interest calculated on a monthly basis) for a thirty year period. If you can come up with a 20 percent down payment, your interest rate will be only 7.75 percent (because you will no longer need to pay for mortgage insurance).
    a. What is your monthly payment if you borrow the full amount?
    PV = 500,000
    Annual interest = 8, but monthly interest = 8/12 = 0.67
    Since you're paying the loan off over the period, the future value of the obligation is 0
    Making monthly payments over 30 years so n = 30*12 = 360
    PMT = ?
    On your calculator, 1) key in 500000 and press PV key; 2) key in 8/12 and press the = key then press the I/Y key; 3) key in 0 and press FV key; 4) key in 30 X 12 and press = key then press N key; 5) to get answer press CPT key and then press PMT key. You should see 3,668.82 which will be your monthly payment. (Note: the calculator will present this answer as a negative because the payment represents an outflow of money because you entered the loan amount as an inflow of money. From the bank's perspective, they would enter the loan amount as -500,000 since they're giving up money, but the payment calculated would then be a positive amount/inflow).
    b. What is your monthly payment if you can come up with the 20 percent down payment?
    Everything is the same as above except your loan amount which is now $500,000 – (0.20*500,000) = $400,000 and your monthly interest rate is 7.75/12 = 0.64583
    On your calculator, 1) key in 400000 and press PV key; 2) key in 7.75/12 and press the = key then press the I/Y key; 3) key in 0 and press FV key; 4) key in 30 X 12 and press = key then press N key; 5) to get answer press CPT key and then press PMT key. You should see 2,865.65 which will be your monthly payment (again as a negative).
    c. What things should you consider when deciding whether to make a down payment or not?
    Obviously cash on hand is a practical consideration. However, other considerations would include how good of an interest rate the bank is offering you. If the bank's rate is significantly lower than the rates on

your other debt, you're better off not making a down payment and instead paying down the higher cost debt. Similarly, if your investments make a rate of return higher than the bank's interest rate, you may be better off not making the down payment and instead investing the money. Also, keep in mind that mortgage interest is tax deductible while most other debt is not, so the rate to consider when comparing this interest charge to other uses of the money is the after tax interest rate. The value of the deduction will be higher the higher your marginal tax rate is. Other issues?

2. Before you purchase the house described above, you find out that the cheapest house in your managing partner's neighborhood is up for sale for $750,000. You really think it would be smart to buy it but the monthly payment from the problem above is about the most you could possibly spend (given your income and other obligations) and you have no outside borrowing options (i.e., your parents aren't in a position to loan you money for a bigger down payment). Your realtor tells you that he knows of a bank that offers "interest only" loans (i.e., your monthly payment is only for interest; there is no principal component) at very low rates. You check it out, and the bank will loan you $750,000 at 6 percent annual interest for 10 years but you need to pay back the full amount of the loan at the end of the period.
   a. What will your monthly payment be under this interest only loan?
   You may intuitively see that under an interest only loan, the balance never declines, so you are always simply paying the monthly interest (i.e., 6/12 = 0.5%) on the original balance. You could figure this out without a financial calculator by simply calculating 750,000*0.005 = $3,750.
   If you were going to use the calculator though, you would simply need to recognize that PV = 750,000 (money you get from the bank) and FV = -750,000 (money you pay back to the bank). I = 6/12 = 0.5; n = 10*12 (though actually the time period does not affect the payment since you're always just paying interest on a non-declining balance):
   On your calculator, 1) key in 750000 and press PV key; 2) key in 6/12 and press the = key then press the I/Y key; 3) key in -750000 and press FV key; 4) key in 10 X 12 and press = key then press N key; 5) to get answer press CPT key and then press PMT key. You should see 3750 which will be your monthly payment (again as a negative because it's an outflow).
   b. What factors should you consider before you go the interest only route?
   At the end of the loan you will need to pay back the full purchase price all at once so you had better have the financial wherewithal to do so, or have the expectation that your house appreciates (at least fast enough to cover the depreciation of the house's condition) over the time period so that you can sell (or borrow through another mortgage) for an amount at least as high as the original purchase price. On the positive side, interest only loans allow you to keep your money freed up if you believe you have investment opportunities making a better return than the after-tax interest rate on the interest only loan.

3. You want to retire in 10 years. You estimate that you can live frugally in your retirement for about $2,000 per month (assume no inflation). You also expect to live for exactly 20 years after you retire.
    a. How much will you need to have available at the end of 10 years to purchase an annuity that will provide for your needs, assuming you can find an annuity provider willing to guarantee you a 5 percent annual rate of return?

    FV = 0 because you expect to die at the end of the period; PMT = 2,000; I = 5/12; n = 20*12; PV = ?
    On your calculator, 1) key in 5/12 and press the = key then press the I/Y key; 2) key in 0 and press FV key; 3) key in 20 X 12 and press = key then press N key; 4) key in 2000 and press PMT key; 5) to get answer press CPT key and then press PV key. You should see 303,050.63 which will be the cost of your annuity (it will be a negative number because it will be a cash outflow for you).

    b. Assuming you expect to make an average annual return of 9 percent in your portfolio, how much should you save per month during the next 10 years?

    You need to have an amount equal to the cost of your annuity at the end of 10 years so FV = 303,050.63; you are going to pay into this, so we assume you put no money down so PV = 0; I = 9/12 = 0.75; n = 10 *12 = 120; PMT = ?
    On your calculator, 1) key in 303,050.63 and press FV key; 2) key in 9/12 and press the = key then press the I/Y key; 3) key in 0 and press PV key; 4) key in 10 X 12 and press = key then press N key; 5) to get answer press CPT key and then press PMT key. You should see 1,566.04 which will be how much you need to save every month to have enough money to buy the annuity in 3a.

    c. Assume you saved as suggested in 3.b, but in the meantime life expectancy has grown significantly and is seemingly growing without bound. You are very fearful of running out of money, but you still want to retire. Your annuity company offers you perpetuity with a 4 percent annual rate of return that is transferable to your heirs upon death. What monthly income stream will this perpetuity provide you?

    Remember that the value of a perpetuity is equal to the payment/interest rate. That means that the income stream is equal to the interest rate times the value. So:
    Key in 303,050.63*monthly interest rate = 303,050.63*(.04/12) = 303,050.63*0.0033 = 1,010.17 which is how much money you and your heirs will get per month forever.

4. Your rich uncle dies and leaves you a fortune of $80 million. You immediately quit your job as a junior associate to join Paris Hilton on the New York party scene. Assuming you take your inheritance and invest it in a Fidelity mutual fund with a constant 7 percent annual rate of return and you spend $500,000 per month.

a. At what point will you be broke?

PV = 80,000,000; FV = 0 (since the question asks at what point you won't have any more money); I = 7/12; PMT = -500,000 (spending is a cash outflow); n = ?

On your calculator, 1) key in 80000000 and press PV key; 2) key in 7/12 and press the = key then press the I/Y key; 3) key in 0 and press FV key; 4) key in -500000 and press PMT key; 5) to get answer press CPT key and then press N key. You should see 465.59 which will be how many months this spending is sustainable for (i.e., 38 years 9 months and 18 days).

b. What if your mutual fund only makes a 5 percent return?

All is same as above except now I = 5/12; HINT: if some of your variables stay the same, they are still in your calculator's memory so you can simply key in the new interest rate and then press CPT and N to get the new answer which is 264.22 (22 years and 7 days).

c. If you want your money to last for 50 years and you conservatively estimate you will make a 4 percent rate of return, how much can you spend per month?

What changes in this problem? Now N=50*12=600 and I = 4/12 and PMT = ?. From 4b you just need to change the N and press CPT and PMT to get 308,565.60 (in a negative because it represents your sustainable monthly cash outflow).

5. You win the lottery. The state gives you two payment options. It will either give you a lump sum payment of $20 million today, or it will pay you $132,000 per month for the next 20 years.

a. What rate of return makes the two options equivalent?

So we know that for equivalence to hold PV = 20,000,000; PMT = 132,000; N = 20*12=240; FV = 0; I=?.

On your calculator, 1) key in 20000000 and press PV key; 2) key in 20 X 12 and press the = key then press the N key; 3) key in 0 and press FV key; 4) key in -132000 and press the PMT key; 5) to get answer press CPT key and then press I/Y key. You should see 0.41673 which will be your monthly interest rate which is an annual rate of 0.41673*12 = 5%.

b. How much more money could you draw per month (for the 20 year period) if you took the lump sum payment and invested it in a fund that paid 8 percent annually (assuming you exhaust the money in 20 years)?

Now I = 8/12 and PMT = ? so you can simply key in 8/12 and press the = key and then the I/Y key then press CPT key and then the PMT key and you should get 167,288.01 which is the monthly draw you could get if you take the lump sum and then invest it in the way described.

c. If you are the official running the state lottery and you need to decide on the terms at which the lottery pays out its jackpots (specifically, what rate of return to use in calculating monthly payments), what factors should you consider in your decision?

What interest rate is the state getting on its investments/paying on its debt. If the state uses a higher interest rate in its calculation of the

annuity payments, more people will let the state hold onto the money. That money can then be used to invest/pay down debt. Also, the state may worry that individuals do not save enough so, by enticing individuals to take the annuity rather than the lump sum, the state can implicitly encourage saving on the part of lottery winners.

Problem Shooting: If you kept getting goofy answers even after following the step by step instructions then your settings are different than mine are. That's OK but you need to keep it in mind when doing problems. Specifically if the calculator is expecting a monthly interest rate and you key in an annual rate, you're going to get different answers.

I have my setting set in the following way – If I press the 2nd key and then the I/Y key (so I can access the setting for P/Y i.e., periods per year) I key in 1.000000 ENTER. With this setting if I want to figure out monthly payments, I know to enter N as the number of months and I/Y as the monthly interest rate. If I want to figure out problems where interest is only calculated once per year, I enter the yearly interest rate for I/Y and N as number of years. I like this kind of setting because I know that, for whatever compounding rate I'm interested in (daily, monthly, quarterly, yearly, etc), I simply need to make sure that N and I/Y match up – that is, if I'm interested in quarterly compounding, I enter the number of quarters for N and the quarterly interest rate; if I'm interested in daily compounding, I enter the number of days as N and the daily rate as I/Y. Everything stays consistent.

However, if I had the P/Y set to 12, the calculator is going to automatically going to divide the interest rate I enter for I/Y by 12. That's fine (and may even be convenient) but I'll be entering an annual interest rate but the N I enter will be in number of months (you can change that too with the xP/Y key). If all the problems you do are all the same kind (everything's monthly), this may be more convenient. But if you then switch the kind of problem you're doing (now, say, daily compounding), you have to reset your settings or do some mental arithmetic such as converting the annual rate you're given into a daily rate then multiplying it by 12 to have a number to enter for I/Y that the calculator will then divide by 12 (arghhhhh) . . . you get the picture. Just have the settings set in the way that's most convenient for you, but I find P/Y = 1 is easiest for me and it's how I'll spell out solutions in the answer keys.

Problem Sets 2&3
Finance
Klick

1. **You represent a professional baseball pitcher who has lost the ability to use his pitching arm due to the negligence of a surgeon in the course of routine rotator cuff surgery during the off-season. You seek lost compensation for the remainder of the pitcher's expected career. The pitcher's last contract paid him $5,000,000 per season. A reasonable estimate is that, but for this mishap, your client would have gone on to pitch another 12 seasons.**
    a. **Assuming a 5% annual discount rate, what amount of compensatory damages will you seek?**

       ANSWER: $44,316,258.18

    b. **The state in which you are suing allows medical malpractice claims to be paid out in equal annual installments over a 20 year period mandating a 3% interest rate by statute. If the defendant chooses to pay out the amount you calculated in 1.a in installments, what annual payment will your client receive?**

       ANSWER: $2,978,748.65/YEAR

    c. **Given the difference between your client's 5% discount rate and the mandated interest rate in the statute (3%), how much more does your client value the lump sum payment from 1.a than the annuity payment from 1.b?**

       ANSWER: ANNUITY IS WORTH $37,121,792.23 SO DIFFERENCE IS $7,194,465.95

2. **You succeed in the lawsuit described above, receiving the lump sum payment described in 1.a . Your client asks you for financial advice; he wants to know how much money he can spend every year without exhausting his resources before he dies.**
    a. **If you assume he will live another 50 years, and can expect a constant 5% return on his savings, how much can your client spend each year?**

       ANSWER: $2,427,499.95/YEAR

    b. **If he will live another 60 years, and earns 5% on his savings, how much can he spend?**

       ANSWER: $2,341,147.47/YEAR

c. **If he says he plans to spend $3 Million per year, how long can he live before he goes broke (assuming a 5% rate of return)?**

ANSWER: 27.5 YEARS

d. **He tells you that he plans to spend $3 Million per year for the next 50 years at which point he plans to die. He also says he wants to leave $100 Million to his children upon his death. For this to be possible, what return must he expect to make on his savings?**

ANSWER: 7.07%

3. **Your client comes back to you 15 years after you worked out a financial plan for him to achieve his wishes as expressed in 2.d .**
   a. **He tells you that his children have turned out to be rotten and so he wants to leave them with nothing upon his death. He wants to know how much he can now spend to achieve his goal of dieing penniless assuming a rate of return of 7% (in the past and for the future) on his savings.**

   ANSWER: $3,620,951.08/YEAR

   b. **He begins to feel guilty and decides that he should provide for his children and subsequent grandchildren; however, they cannot be trusted to invest their inheritance wisely, so, upon his death, he wants to purchase each of his three children a perpetuity that will pay $1 Million per year. Again assuming a 7% interest rate, how much money does your client need to have remaining upon death to purchase the perpetuities?**

   ANSWER: $42,857,142.86

   c. **Does he need to recalculate his spending habits?**

   ANSWER: HE CAN NOW SPEND UP TO $3,310,924.24/YEAR AND STILL HAVE ENOUGH TO BUY THE ANNUITIES UPON HIS DEATH 35 YEARS FROM NOW

4. **You wish to value a bond with the following properties:**
   a. **Coupon payment of $1,000/year for 3 years. Face value of 20,000. Assume that your discount rate for the first period is 2%, second period is 4%, and third period is 5%. What are you willing to pay for the bond?**

ANSWER: FIRST PERIOD IS WORTH $980.39; SECOND PERIOD IS WORTH $924.56; THIRD PERIOD IS WORTH $18,140.59 . Bond is worth $20,045.54 .

b. **Assuming your valuation is the same as the market's, is this bond selling at a premium or a discount?**

ANSWER: It's selling at a premium

c. **Assume that this bond is a zero coupon bond (i.e., you get $20,000 at the end of 3 years). What are you willing to pay for the bond now?**

ANSWER: $17,276.75

5. **You work for an investment bank that needs to price shares of a private firm that is going public.**
   a. **Given the number of shares the firm wants to issue, a good estimate of earnings suggests that earnings per share will be $5 next year. A reasonable discount rate, given the riskiness of this firm, is 20%. If you price the shares at $33, what growth rate are you assuming for this firm's earnings?**

   ANSWER: 4.85%

   b. **If you think that the firm's earnings will grow at 10% per year, and you still price the stock at $33/share, what is the implied discount rate?**

   ANSWER: 25%

   c. **How do you go about determining the proper discount rate in a valuation problem such as this?**

   ANSWER: DEPENDS ON YOUR OPPORTUNITY COST OF FUNDS AND THE RISKINESS OF THE ASSET

6. **Value a stock under the following assumptions:**
   a. **It has an implied dividend of $3/share that is expected to grow at 5%/year, and your discount rate for the stock is 10%. You plan to hold the stock forever.**

   ANSWER: VALUE = DIVIDEND/(i-g) = 3/(.1-.05) = $60

   b. **Instead, assume you plan to hold the stock for three periods and then sell. Your estimate of the relevant discount rate and growth rate matches that of the market in general, and your growth rate**

prediction proves to be correct over the relevant time frame. How much are you willing to pay for this stock.

You will get $3 in year one, 3*1.05 = $3.15 in year 2, and 3.15*1.05 = $3.308 in year 3. You will then sell the stock to someone who values it by calculating the next period's dividend (3.308*1.05 = 3.473) and plugs it into his perpetuity formula 3.473/.05 = 69.46.

Now when you go about valuing these income streams, you discount all the payments and add them up 2.73+2.60+2.49+52.19 = $60

c. Explain the relationship between your answers in 6.a and 6.b .

As I said in class, you can break down any perpetuity into an annuity for a term and then a sale of the perpetuity, so, for valuation's sake, it doesn't matter how long you hold the stock (ex ante) since even if you sell it, the person who you expect to buy it will value it as infinitely lived (or he will value it as an annuity plus a sale of an infinite income stream in the future to someone who . . . you get the picture).

Problem Set 4
Finance -- Klick
March 21, 2005

1. Compute the variance, standard deviation and mean of each of the following series; also graph them to gain an appreciation of what a more dispersed series looks like:

A.

12	5	8	4	3	8	15	6	4	2

B.

1	11	10	1	13	2	15	3	1	10

C.

7	6	7	8	6	5	8	6	6	8

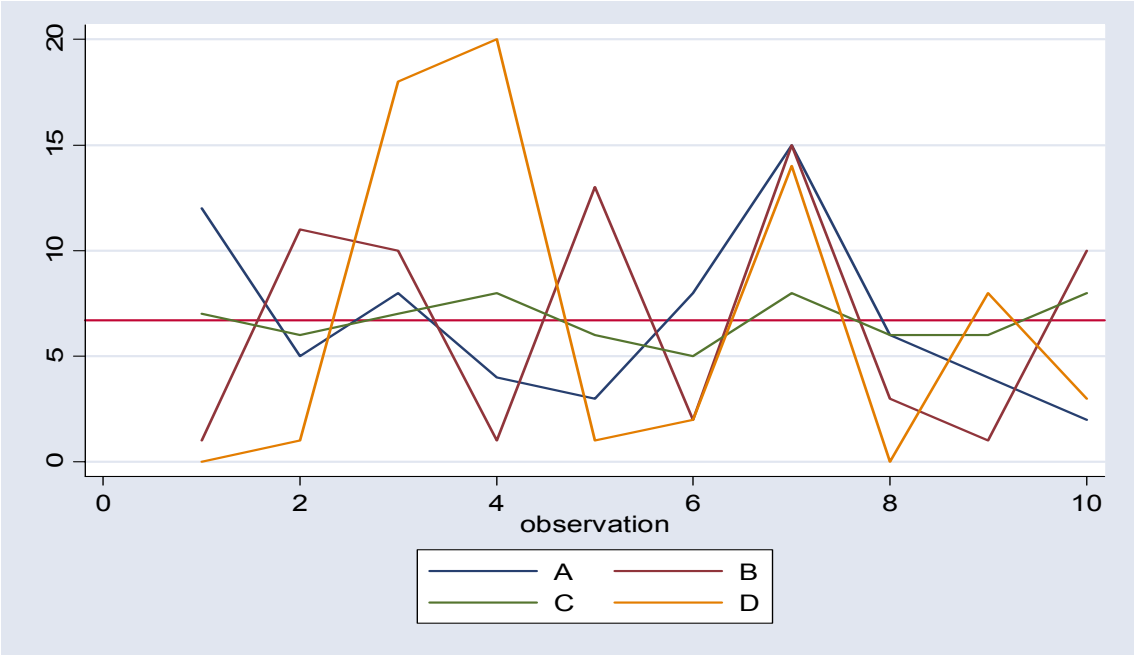
D.

0	1	18	20	1	2	14	0	8	3

E.

2	4	6	15	8	3	4	8	5	12

Series	Variance	Standard Deviation	Mean
A	15.41000	3.92556	6.7
B	28.21000	5.31131	6.7
C	1.01000	1.00499	6.7
D	55.01000	7.41687	6.7
E	15.41000	3.92556	6.7



Note that E is just A in reverse so I didn't graph it.

2. The following returns represent the dollar gains from a given portfolio for a ten year period:

5000 8000 -2500 1000 -3000 6000 4500 -9000 500 7500

Calculate the mean of the return and the variance of your dollar return over this period under the following assumptions: a) You own the entire portfolio; b) you own 1/10 of the portfolio; c) you own 1/100 of the portfolio; d) you own 1/1000 of the portfolio.

Share	mean	Variance
Whole	1800	26660000
1/10	180	266600
1/100	18	2666
1/1000	1.8	26.66

Use the answers you calculate in 2.a – 2.d to convince yourself that variance declines faster in relative terms than does mean as you decrease your ownership stake in the portfolio (i.e., as you move from a personal portfolio to equivalent "mutual funds" with smaller ownership stakes).

As you can see, as you move down the rows, your mean return decreases by a factor of 10, while your variance decreases by a factor of 100.

3. You are trying to decide between buying security A and security B, each of which cost $1,000. Security A is a low risk security such that, if the economy worsens next year, it will be worth $900 at the end of the year; if the economy grows at the current rate, it will be worth $1,050 at the end of the year; if the economy grows faster than normal, it will be worth $1,100. Under the 3 scenarios, security B will be worth $0, $1,500, and $2,000. Your best estimate suggests that there is a 30% chance that the economy will worsen; a 60% chance the economy will remain unchanged; and a 10% chance the economy will grow more quickly than normal next year.

a. What is the your estimate of what each security will be worth next year?

Security A: Value = .3*900+.6*1050+.1*1100 = 270+630+110 = 1010
Security B: Value = .3*0+.6*1500+.1*2000 = 0+900+200 = 1100

b. What is the variance of each security's value, given your predictions of the economy's prospects? [Note, the variance formula we had in class gave each observation an equal weight; in this case, you are not calculating the variance over n equal observations. Instead, you are calculating the variance over 3 states of the world that are not equally likely to come about. Therefore, you are not doing an average (i.e., 1/n), but rather a weighted average of the squared deviations from the mean.]

Variance A = .3*(900-1010)^2+.6*(1050-1010)^2+.1*(1100-1010)^2 = 5,400
Variance B = .3*(0-1100)^2+.6*(1500-1100)^2+.1*(2000-1100)^2 = 540,000